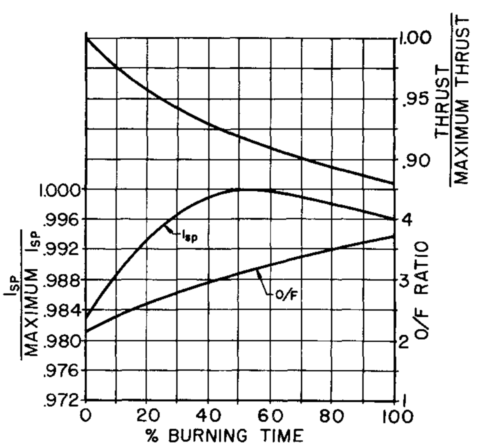

Fig. 3 Motor performance.

oxidizer injection rates, resulting in O/F ratios of about 1.2 to 3.8 at minimum and maximum thrust, respectively. Referring again to Fig. 2, it is easy to see the reduction in specific impulse at the low O/F ratios and reduced chamber pressure. However, the major effect of this wide O/F range is in propellant utilization. With a fixed amount of propellant provided, 75% of which is oxidizer, continuous low-thrust operation at less than the design O/F would result in an excess of oxidizer; e.g., 61% of the oxidizer would remain after operation at $O/F = 1.2$. Continuous high-thrust operation would not utilize all of the fuel available. Therefore, any thrust level other than that corresponding to near optimum O/F ratio reduces effective mass fraction as well as the specific impulse of the propulsion system. For the vehicle assumed in this analysis, the accumulated effect of performance degradation on velocity is shown in Fig. 4 as a function of throttling ratio.

Operation at a constant O/F ratio at or near optimum can be achieved by utilizing aft-end make up in the hybrid motor design (Fig. 5). Oxidizer injectors are provided at both ends of the chamber. At the head end, sufficient oxidizer is sprayed into the fuel-lined chamber to produce the required fuel-weight flow rate for any selected thrust level. The remaining oxidizer to provide the optimum mixture ratio is injected at the aft end of the fuel grain just upstream of the nozzle. The use of this technique permits maintenance of an optimum O/F ratio at any thrust level. A better understanding of what occurs is obtained by examining the equations presented earlier. A low level of oxidizer flow is required to provide a low level of thrust. A low chamber pressure also results, but this affects only the I_{sp} and not the

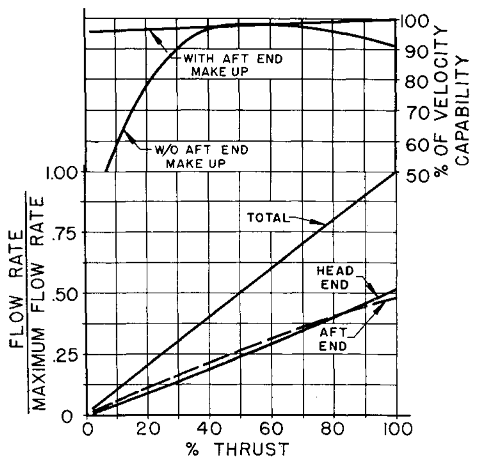

Fig. 4 Vehicle performance and flow rates.

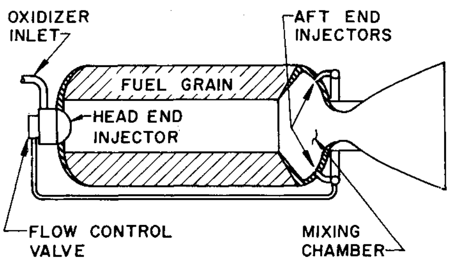

Fig. 5 Hybrid engine schematic.

regression rate. The oxidizer injected aft of the solid fuel grain has no effect on the fuel flow and thereby only increases the O/F ratio. Proper liquid oxidizer flow control then can result in optimum O/F ratio operation. A single flow-control valve can be utilized to provide the proper amounts of oxidizer flow to the injectors at each end of the engine. The relative total oxidizer flow rate and the oxidizer flow rate at each end are shown in Fig. 4 as functions of the percentage of thrust desired for a typical design. The aft-end make-up technique is considered to be a practical solution to the problems involved in wide throttling of a hybrid engine.

Conclusions

Achieving constant thrust in a hybrid engine requires a varying oxidizer flow rate and results in some degradation in performance due to degraded specific impulse. Nearly constant thrust is easier to achieve through the use of a constant oxidizer flow rate and with only slight performance penalties. Wide throttling requires a unique design solution for both thrust control and maximum propellant utilization. The aft-end make-up technique provides this solution.

Liquid Frequencies and Damping in Compartmented Cylindrical Tanks

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DEMANDS for increasing the payload in space missions have led to larger and longer launch vehicles and hence to fuel and oxidizer tanks of enormous capacity and, unfortunately, very low propellant slosh frequencies. Such low frequencies are undesirable because of the likelihood of coupling with structural modes or the autopilot system. Clustering smaller diameter tanks has offered a means of providing large volume with substantially higher liquid resonant frequencies than in a single tank for a vehicle with the same over-all fineness ratio. Compartmentation of a single cylindrical tank into sectors also provides a means of shifting the liquid resonant frequencies.

The present note presents data on liquid resonant frequencies and damping coefficients in a compartmented tank undergoing forced translational oscillation. Flat-bottomed cylindrical tanks were compartmented into equal sectors of 90°, 60°, or 45°. Materials for the sector walls varied from solid to perforated stocks with various hole diameters and

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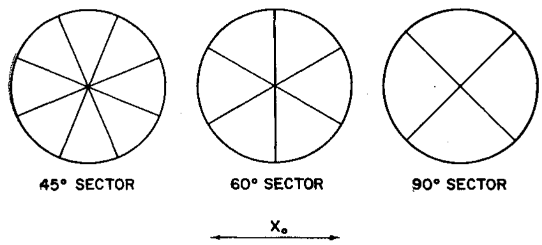

Fig. 1 Tank configurations.

percent openings. The tests were conducted under a gravity axial acceleration field and at a liquid depth equal to the tank diameter ($h/d = 1$). The tanks were force-excited through a frequency range that included the two lower resonant frequencies.[1–3] The direction of translational excitation relative to the sector walls is as shown in Fig. 1. All the tests were conducted for three values of translational excitation amplitude (x_0/d), using the same facility and methods employed in Ref. 4.

Liquid Resonant Frequencies

Natural frequencies for liquids in sectored cylindrical tanks were computed by the method of Refs. 1 and 2. However, the effect of translational excitation amplitude on the natural frequencies cannot be accounted for in the computations, and it is this excitation amplitude that causes a significant difference between the theoretical and experimental natural frequencies. Previously exploratory experiments in compartmented cylindrical[1] and in spherical tanks[5] have indicated that the theoretical natural frequencies can be approached only for small values of excitation amplitude ($x_0/d \leqq 0.00187$); for values greater than this, a significant decrease in natural frequencies exists with increase in excitation amplitude.

The liquid natural frequencies presented in this note are the lowest resonant frequencies measured for the 90°, 60°, and 45° compartmented tanks undergoing translational excitation. Figure 2 shows the theoretical natural ($x_0/d = 0$) frequencies and the experimental resonant frequencies vs translational excitation amplitudes for solid-sector walls. The theoretical natural frequencies are approached as the translational excitation is reduced.

Even though perforated ring baffles are widely used for liquid damping,[6] perforation of the sector walls in compartmented tanks can have a very large and detrimental effect on the liquid natural frequencies, far outweighing the advantages to be gained from the increased damping. Figure 3 shows the frequency variations in water in tanks with perforated sector walls for a translational excitation amplitude of $x_0/d = 0.00417$. The dimensionless resonant frequency parameter ($\omega^2 d/a$), where ω is the liquid natural frequency and a is the acceleration field, is plotted against an equivalent Reynolds number based on perforation hole size, translational excitation amplitude, and liquid properties and seems about as effective as any, in view of the over-all complexity of the data. The most important feature in these figures, obviously, is the large decrease in resonant frequency with increasing perforation hole size and percent open areas.

Liquid Damping

The damping at the resonant frequency for the 90°, 60°, or 45° *solid wall* tanks is low, averaging approximately 0.04. However, at frequencies below the resonant value, the liquid sloshing is effectively damped. The tests showed that *perforated* sectors with less than 10% open area will increase the damping ratio to approximately 0.1, while maintaining a liquid resonant frequency corresponding to a solid wall compartmented tank. For partitions with open area greater than 10%, the damping produced is greater than 0.10, but the corresponding liquid resonant frequencies approach that of an uncompartmented cylindrical tank. The liquid viscosity and excitation amplitude have as large an effect on the liquid damping ratios as do the perforation hole size and percent of open area, and as much as all of these factors have on the liquid resonant frequencies.

Attempts to present the damping ratios vs Reynolds number, or other various parameters that include the factors

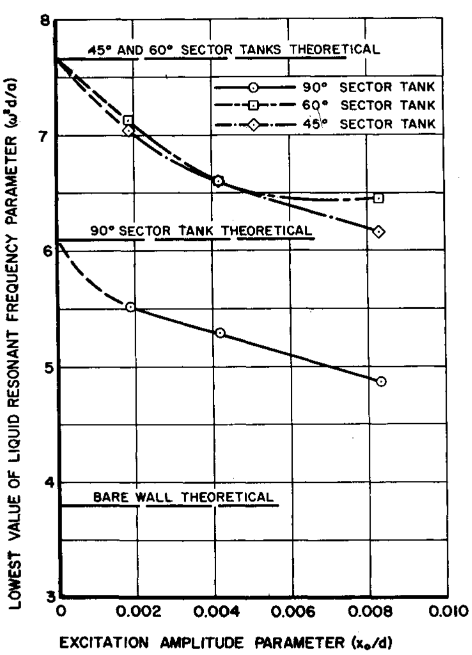

Fig. 2 Effect of excitation amplitude on the lowest resonant frequency for 45°, 60°, and 90° sector tanks.

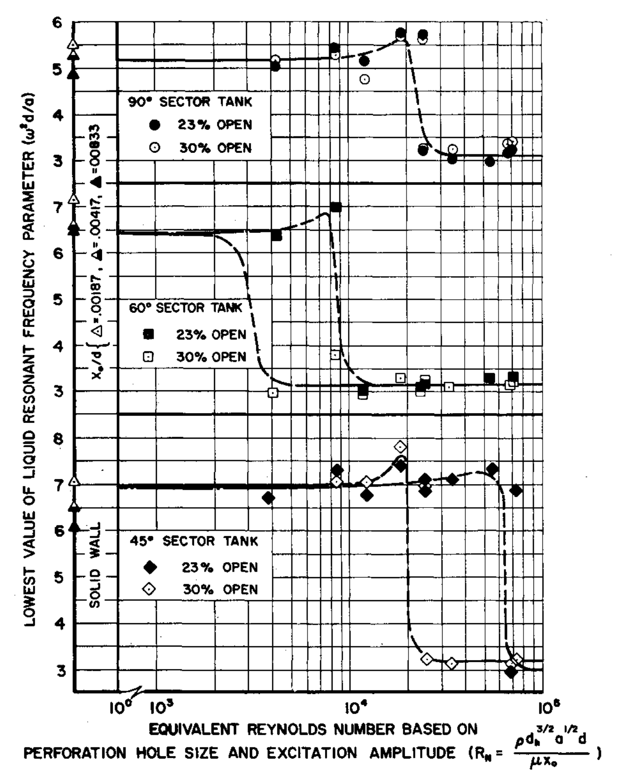

Fig. 3 Variation in lowest liquid resonant frequency with Reynolds number for 45°, 60°, and 90° perforated sector tanks.

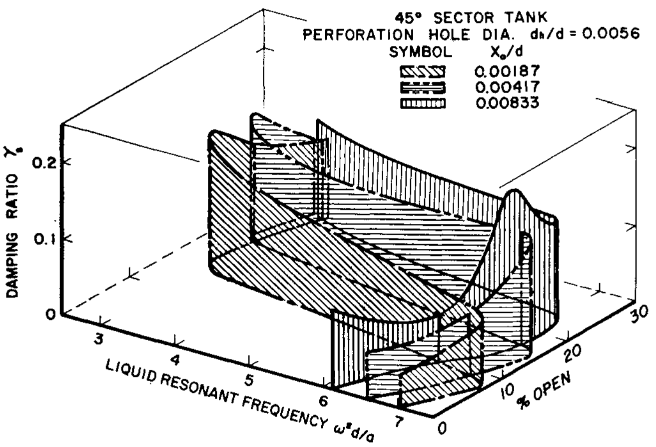

Fig. 4 Variation in damping ratio for 45° sector tank.

affecting the damping, failed to yield an effective picture of this complex data. The best that can be done is something such as that shown in Fig. 4, which is a three-dimensional plot of the damping ratio vs the dimensionless resonant frequency parameter $\omega^2 d/a$ and the percentage of open sector area for a 45° sectored tank. The results presented are for three values of x_0/d, with water and methylene chloride as the test liquids. Maximum damping is produced for sectors with open areas of 16 to 23%. However, the excitation amplitude must be quite large to maintain the resonant frequency corresponding to solid-sector walls. Additional tests with sectors of smaller hole diameter ratios (d_h/d = 0.00139 and 0.00278) and open areas up to 23% increased the damping ratios to an average value of 0.15 while maintaining a frequency corresponding to the solid wall sector tank. Above 23% open area, the results become inconsistent in frequency and damping ratio.

Test results for 60° and 90° tanks with $d_h/d = 0.0056$ are also available. For the 60% sector tank, increased damping ratios at frequencies corresponding to the solid-sector wall exist only for open areas between 8 and 16% open area, depending on the excitation amplitude. Additional tests with $d_h/d = 0.00139$ and 0.00278 yielded mean damping ratios of approximately 0.12 at 16% to 23% open sector area, depending on excitation amplitude. The smaller open area corresponds to the lower excitation amplitudes. In the 90° tank, sectors with $d_h/d = 0.0056$ lose their compartmentation effect at small excitation amplitudes; however, tests with sector walls having $d_h/d = 0.00139$ and 0.00278 and up to 30% open area produced mean damping ratios of 0.12 at frequencies corresponding to the solid sector wall for $x_0/d = 0.00417$ and 0.00833. Tests at $x_0/d < 0.00417$ with $d_h/d = 0.00139$ gave results similar to the solid sector. The results for $d_h/d = 0.0279$ were inconclusive, and no consistent damping ratios or resonant frequencies were obtained.

Conclusions

Compartmentation of cylindrical tanks with *solid* walls is an effective means of increasing the lowest resonant frequency of the liquid. For large values of translational excitation amplitude, a significant decrease in resonant frequency exists. The liquid damping ratio with these solid-sector wall tanks averaged approximately 0.04. Compartmentation with *perforated* walls show less sensitivity to translational excitation amplitude, particularly for the tank with 45° sectors. However, for lower amplitudes, the resonant frequency decreased to a value that approximated the resonant frequency of an uncompartmented cylindrical tank for the three-sectored configurations. At the compartmented resonant frequencies, the liquid damping ratios were about 0.08 to 0.1. When the resonant frequencies approximated those attained in the uncompartmented tank, the highest liquid damping ratio increased to 0.2.

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Interior Ballistics of Spinning Solid-Propellant Rockets

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Nomenclature

V = velocity
ω = angular velocity
r = radial distance from axis
D = diameter of motor
ϕ = parameter defined in Eq. (5)
a = sonic velocity
A = cross-sectional area
θ = spinning parameter defined in Eq. (10)
P = pressure
n = burning rate-pressure index
t_b = duration of burning

Superscripts

$(\ )^*$ = conditions at nozzle throat
$(\ )'$ = properties measured in spinning-coordinate system (unprimed properties measured in fixed coordinate system)

Subscripts

$(\ )_0$ = conditions at propellant burning surface
$(\ )_a$ = properties measured in axial direction
$(\ )_t$ = properties measured in tangential direction
$(\ )_\omega$ = ballistic properties of spinning motors

In the development of spin-stabilized projectiles propelled by solid-propellant rocket motors, a number of cases have been observed wherein the apparent burning rates of spinning motors were substantially greater than those in similar motors fired in fixed test stands. Some observers have explained this phenomenon as being a result of erosive burning, but it is difficult to see how erosive burning conditions could be produced by spinning a motor. It is postulated here that the spinning motor establishes a vortex motion in the combustion gases which produces a reduction of the effective area of the

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